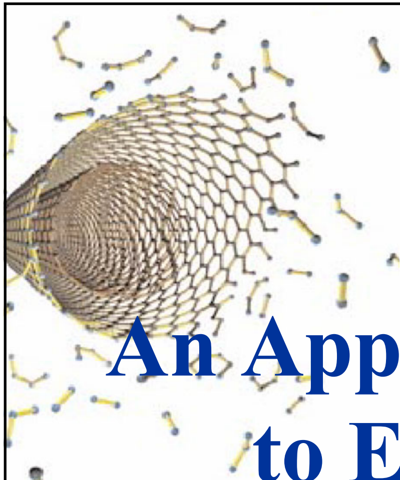


An Application of the meshless BEM to Evaluation of the Thermal Properties of CNT Composites

Masa. Tanaka, J. Zhang, T. Matsumoto

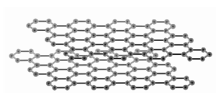
Shinshu University
Faculty of Engineering

Outline

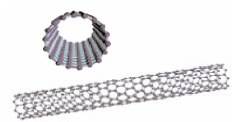
- Introduction
- HBNM for single domain
- Multi-domain HBNM
- Modeling of RVE with single CNT
- Numerical results
- Conclusions

Introduction

- **Thermal conductivity of CNT (W/mK)**


Graphite
50~100


Diamond
3320

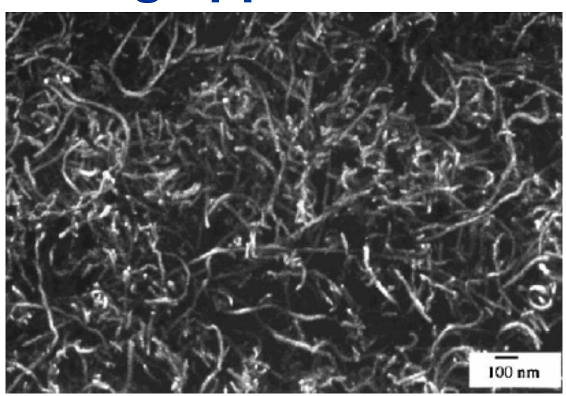

Nanotube
3000~6000

Resins: 0~1 W/mK

Metals:

Fe	72	W/mK
Al	240	W/mK
Cu	390	W/mK

- **Promising applications**



Nanotube-reinforced polymers

HBNM for single domain

- By combining a modified functional with the *Moving Least Squares* (MLS) approximation, the HBNM is a boundary-only truly meshless method
- Three independent variables
  - internal temperature

$$\phi = \sum_{I=1}^{N} \phi_I^s x_I \qquad \phi_I^s = \frac{1}{\kappa} \frac{1}{4\pi r(Q, \mathbf{s}_I)}$$

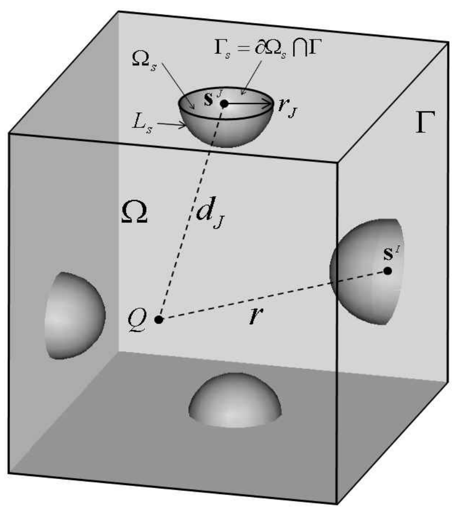
  - Boundary temperature and normal flux

$$\tilde{\phi}(\mathbf{s}) = \sum_{I=1}^{N} \Phi_I(\mathbf{s}) \hat{\phi}_I \qquad \tilde{q}(\mathbf{s}) = \sum_{I=1}^{N} \Phi_I(\mathbf{s}) \hat{q}_I$$

HBNM for single domain (2)

➢ Local weak form

$$\int_{\Gamma}(q-\tilde{q})\delta\phi d\Gamma-\int_{\Omega}\phi_{,ii}\delta\phi d\Omega+\int_{\Gamma_q}(\tilde{q}-\bar{q})\delta\tilde{\phi}d\Gamma-\int_{\Gamma}(\phi-\tilde{\phi})\delta\tilde{q}d\Gamma=0$$



$$\sum_{I=1}^{n}\int_{\Gamma_s}\frac{\partial\phi_I^s}{\partial n}v_J(Q)x_I d\Gamma=\sum_{I=1}^{n}\int_{\Gamma_s}\Phi_I(\mathbf{s})v_J(Q)\hat{q}_I d\Gamma$$

$$\sum_{I=1}^{n}\int_{\Gamma_s}\phi_I^s v_J(Q)x_I d\Gamma=\sum_{I=1}^{n}\int_{\Gamma_s}\Phi_I(\mathbf{s})v_J(Q)\hat{\phi}_I d\Gamma$$

HBNM for single domain (3)

- **System of equations – final form**

$$\boxed{\mathbf{U}\mathbf{x} = \mathbf{H}\hat{\mathbf{q}}}$$

$$\boxed{\mathbf{V}\mathbf{x} = \mathbf{H}\hat{\boldsymbol{\phi}}}$$

where

$$U_{IJ} = \int_{\Gamma_s^J} \frac{\partial \phi_I^s}{\partial n} v_J(Q)\, d\Gamma$$

$$V_{IJ} = \int_{\Gamma_s^J} \phi_I^s v_J(Q)\, d\Gamma$$

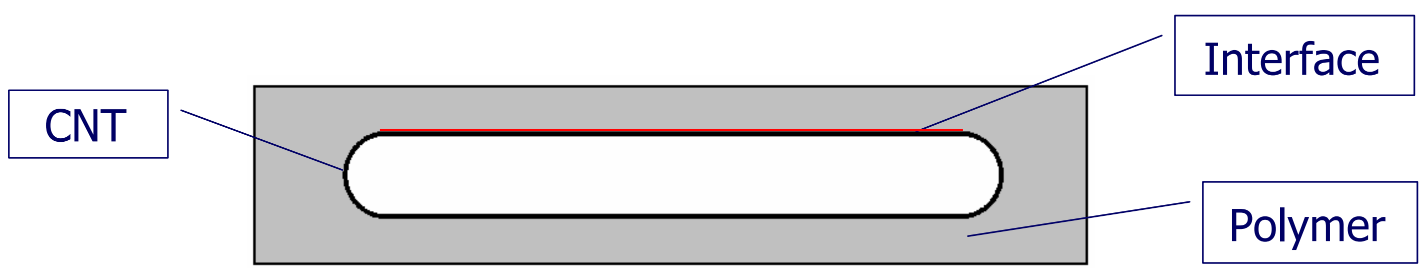
$$H_{IJ} = \int_{\Gamma_s^J} \Phi_I(\mathbf{s}) v_J(Q)\, d\Gamma$$

Multi-domain HBNM

- **CNT and Polymer**

$$\begin{bmatrix} U_{00}^p & U_{01}^p \\ U_{10}^p & U_{11}^p \end{bmatrix} \begin{Bmatrix} x_0^p \\ x_1^p \end{Bmatrix} = \begin{Bmatrix} H_0^p \hat{\phi}_0^p \\ H_1^p \hat{\phi}_1^p \end{Bmatrix} \qquad \begin{bmatrix} V_{00}^p & V_{01}^p \\ V_{10}^p & V_{11}^p \end{bmatrix} \begin{Bmatrix} x_0^p \\ x_1^p \end{Bmatrix} = \begin{Bmatrix} H_0^p \hat{q}_0^p \\ H_1^p \hat{q}_1^p \end{Bmatrix}$$

$$\begin{bmatrix} U_{00}^n & U_{01}^n \\ U_{10}^n & U_{11}^n \end{bmatrix} \begin{Bmatrix} x_0^n \\ x_1^n \end{Bmatrix} = \begin{Bmatrix} H_0^n \hat{\phi}_0^n \\ H_1^n \hat{\phi}_1^n \end{Bmatrix} \qquad \begin{bmatrix} V_{00}^n & V_{01}^n \\ V_{10}^n & V_{11}^n \end{bmatrix} \begin{Bmatrix} x_0^n \\ x_1^n \end{Bmatrix} = \begin{Bmatrix} H_0^n \hat{q}_0^n \\ H_1^n \hat{q}_1^n \end{Bmatrix}$$



Multi-domain HBNM (2)

- **Continuity and equilibrium at the interface**

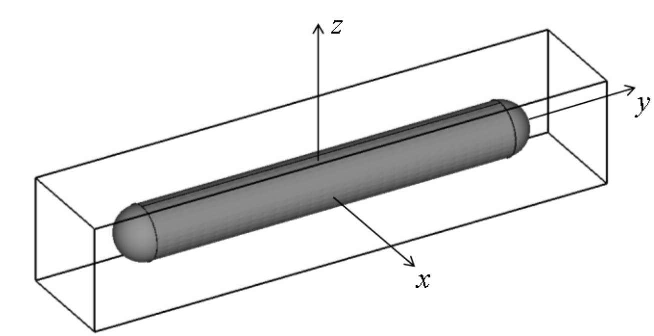
$$\{\phi_1^p\} = \{\phi_1^n\} \qquad \{q_1^p\} = -\{q_1^n\}$$

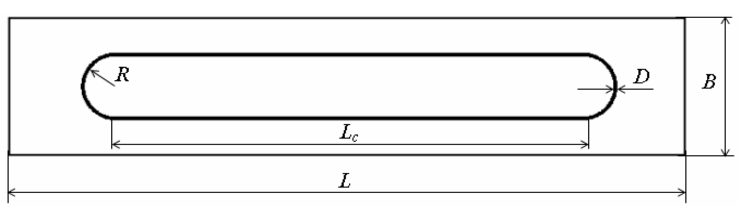
- **Assembled system of equations**

$$\begin{bmatrix} A_{00}^p & A_{01}^p & 0 & 0 \\ U_{10}^p & U_{11}^p & -U_{10}^n & -U_{11}^n \\ V_{10}^p & V_{11}^p & V_{10}^n & V_{11}^n \\ 0 & 0 & A_{00}^n & A_{01}^n \end{bmatrix} \begin{Bmatrix} x_0^p \\ x_1^p \\ x_0^n \\ x_1^n \end{Bmatrix} = \begin{Bmatrix} H_0^p d_0^p \\ 0 \\ 0 \\ H_0^n d_0^n \end{Bmatrix}$$

RVE with single CNT

- **Dimensions and parameters**



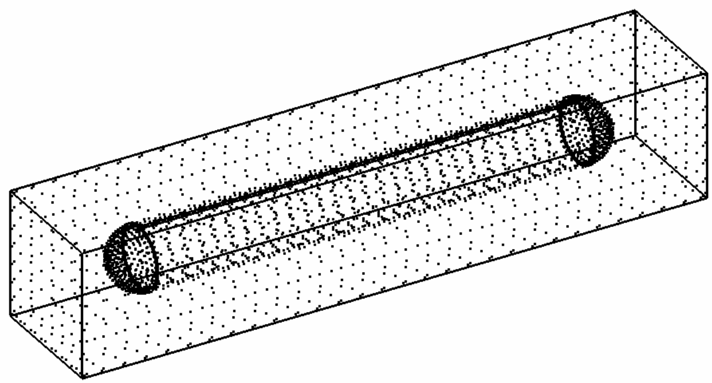


B=20 nm, L=100 nm
L_c=70 nm, R=5 nm
D=0.4 nm

Conductivities
Nanotube: 6000 W/m·K
Polymer: 0.19 W/m·K

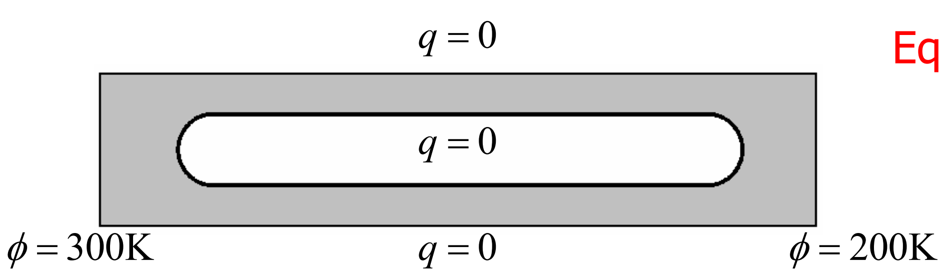
RVE with single CNT (2)

- **Discretization and boundary conditions**



Polymer matrix: 2192 nodes

Carbon nanotube: 2208 nodes

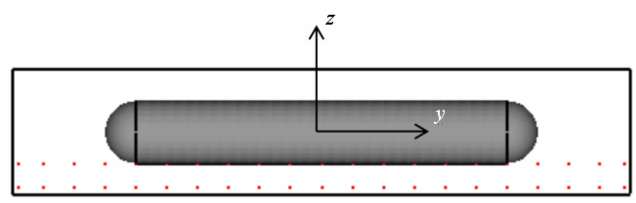


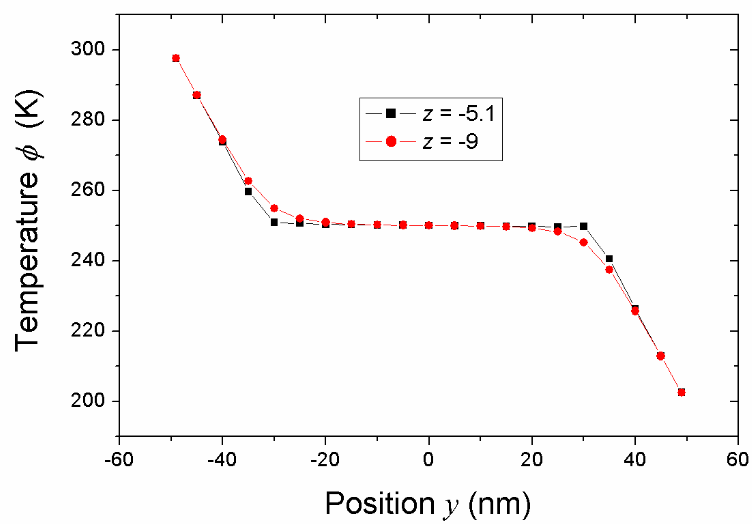
Equivalent heat conductivity

$$\boxed{\kappa = -\frac{qL}{\Delta \phi}}$$

Results

- **Temperature distribution**





Volume fraction:

15%

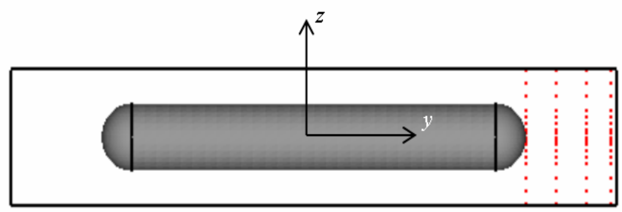
Equivalent conductivity:

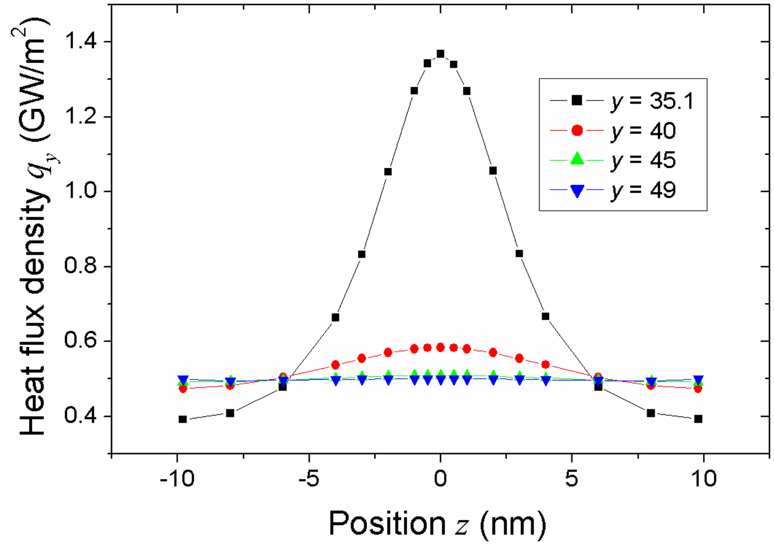
0.6787 W/m·K

(3.57 times that of the matrix)

Results (2)

- **Flux distribution**



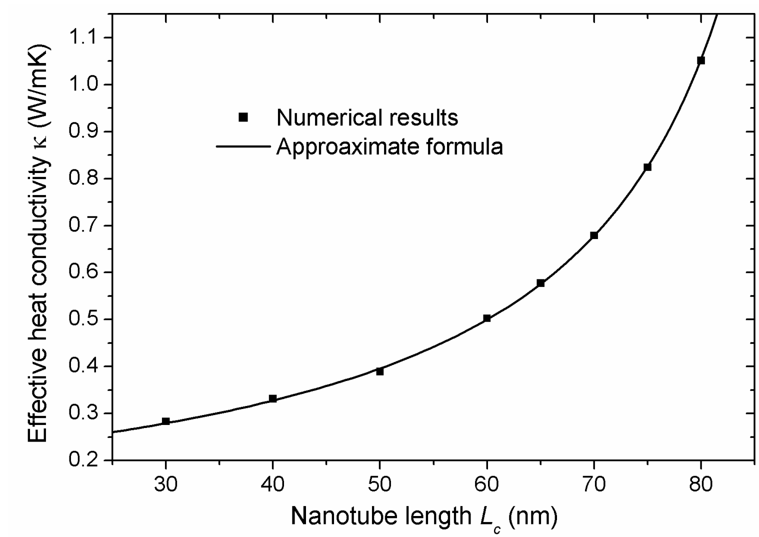


Results (3)

- **Equivalent heat conductivity as a function of CNT length**

$$\kappa^e = \kappa^m \frac{L}{L - L_c - 0.4R}$$

Approximate formula



Conclusions

- The HBNM has been successfully applied to heat conduction analysis of CNT-based composites
- Insight into heat conduction behavior gained:
  - temperature distribution within the CNT is almost uniform
  - heat flux concentration occurs at the tips of the CNT